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DYNAMIC BEHAVIOR OF TWO UNEQUAL PARALLEL PERMEABLE INTERFACE CRACKS IN A PIEZOELECTRIC LAYER BONDED TO TWO HALF PIEZOELECTRIC MATERIALS PLANES *

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Abstract: *The dynamic behavior of two unequal parallel permeable interface cracks in a piezoelectric layer bonded to two half-piezoelectric material planes subjected to harmonic anti-plane shear waves is investigated. By using the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations in which the unknown variables were the jumps of the displacements across the crack surfaces. Numerical results are presented graphically to show the effects of the geometric parameters, the frequency of the incident wave on the dynamic stress intensity factors and the electric displacement intensity factors. Especially, the present problem can be returned to static problem of two parallel permeable interface cracks. Compared with the solutions of impermeable crack surface condition, it is found that the electric displacement intensity factors for the permeable crack surface conditions are much smaller.*

Key words: interface crack; elastic wave; intensity factor; piezoelectric material; Schmidt method

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Introduction

Due to the intrinsic electro-mechanical coupling behavior, piezoelectric materials are very useful in electronic devices. However, most piezoelectric materials are brittle such as ceramics and crystals. Therefore, piezoelectric materials have a tendency to develop critical cracks during the manufacturing and the poling processes. So, it is important to study the electro-elastic

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interaction and fracture behaviors of piezoelectric materials.

The increasing attention to the study of crack problems in piezoelectric materials has led to a lot of significant works. Pak[1] and Chen and Yu[2] studied a single crack problem under the static and dynamic conditions, while Meguid and Wang[3] investigated the dynamic interaction between two cracks in a piezoelectric medium under incident anti-plane shear wave loading. Soh, Fang and Lee[4], and Li and Tang[5] studied the interacting crack problem. Narita, Shindo and Watanabe[6] also examined a piezoelectric layer with an anti-plane shear crack bonded to two elastic half-planes. Zhou and Wang[7,8] investigated the static behavior of two parallel symmetry permeable cracks in piezoelectric materials, and interface cracks in a piezoelectric layer bonded to two half piezoelectric materials planes. To our knowledge, the dynamic electro-elastic behavior of two unequal parallel permeable interface cracks in a piezoelectric layer bonded to two half piezoelectric materials planes has not been studied.

In present paper, the dynamic Mode-Ⅲ crack problem for the interaction between two unequal parallel interface cracks subjected to the harmonic anti-plane shear waves in piezoelectric materials is considered by use of the Schmidt method (Morse[9], Yan[10]). By using the Fourier transform, two pairs of dual integral equations are obtained, and they can be solved by use of the Schmidt method. Some numerical results are presented graphically to show the effects of the geometric parameters, the frequency of the incident wave on the dynamic stress intensity factors and the electric displacement intensity factors.

1 Problem Statement

It is assumed that there is a sort of piezoelectric interlayer with $2h$ in thickness, and which is bonded by another two half piezoelectric material planes. Two parallel interface cracks with different lengths $(2a, 2b)$ are located along the bonding line. A Cartesian system (x, y, z) is positioned with its origin at the center between the cracks for reference purpose. Note that the z-axis is oriented in the poling direction of the piezoelectric materials, and the x-y plane is the transversely isotropic plane, $x = 0$ is a plane of geometric symmetry. To simplify the calculation, in this paper, note superscript k ($k = 1,2,3,4$) refers to the upper half plane 1, the layer 2, the layer 3 and the lower half plane 4 as shown in Fig.1, respectively. It is also assumed that the material of the upper half plane 1 is the same as the material of the lower half plane 4, the material of the layer 2 is the same as the material of the layer 3. In this paper, the harmonic anti-plane shear wave is vertically incident. The mechanical field corresponding to a steady state incident elastic wave can be expressed in terms of the frequency ω, such that $\tau_{yz}(x, y, t) = \tau_0 e^{-i\omega t}$. For the sake of convenience, the time dependence of all field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed and we only consider that τ_0 is positive.

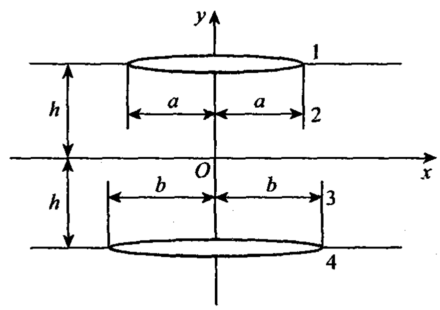

Fig.1 Two unequal parallel interface cracks in piezoelectric materials

The dynamic anti-plane governing equations for piezoelectric materials are given by

$$c_{44}^{(k)} \nabla^2 w^{(k)} + e_{15}^{(k)} \nabla^2 \phi^{(k)} = \rho^{(k)} \frac{\partial^2 w^{(k)}}{\partial t_2}, \tag{1}$$

$$e_{15}^{(k)}\nabla^2 w^{(k)} - \varepsilon_{11}^{(k)}\nabla^2\phi^{(k)} = 0, \tag{2}$$

in which $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian operator, $w^{(k)}$ and $\phi^{(k)}$ are the mechanical displacement and the electric potential, while $\rho^{(k)}$, $c_{44}^{(k)}$, $e_{15}^{(k)}$ and $\varepsilon_{11}^{(k)}$ are the mass density, shear modulus, piezoelectric coefficient and dielectric parameter of the materials, respectively.

The constitutive equations of the piezoelectric materials are

$$\tau_{jz}^{(k)} = c_{44}^{(k)} w_{,j}^{(k)} + e_{15}^{(k)}\phi_{,j}^{(k)}, \tag{3}$$

$$D_j^{(k)} = e_{15}^{(k)} w_{,j}^{(k)} - \varepsilon_{11}^{(k)}\phi_{,j}^{(k)}, \tag{4}$$

where $\tau_{jz}^{(k)}$, $w_{,j}^{(k)}$, $\phi_{,j}^{(k)}$ and $D_j^{(k)}$ $(j = x, y)$ are the stress, strain, electric field strength and electric displacement tensors, respectively. The wave velocity is defined as $c^{(k)} = \sqrt{\mu^{(k)}/\rho^{(k)}}$, where $\mu^{(k)} = c_{44}^{(k)} + (e_{15}^{(k)})^2/\varepsilon_{11}^{(k)}$. Owing to the symmetry in geometry and loading, it is sufficient to consider only the right side of the planes $x \geqslant 0$, $|y| < \infty$.

The boundary conditions of the problem can be stated as follows:

$$\begin{cases} \tau_{yz}^{(1)}(x,h,t) = \tau_{yz}^{(2)}(x,h,t),\ \phi^{(1)}(x,h,t) = \phi^{(2)}(x,h,t), \\ D_y^{(1)}(x,h,t) = D_y^{(2)}(x,h,t), \end{cases} \quad x > 0; \tag{5}$$

$$\begin{cases} w^{(2)}(x,0,t) = w^{(3)}(x,0,t),\ \tau_{yz}^{(2)}(x,0,t) = \tau_{yz}^{(3)}(x,0,t), \\ \phi^{(2)}(x,0,t) = \phi^{(3)}(x,0,t), D_y^{(2)}(x,0,t) = D_y^{(3)}(x,0,t), \end{cases} \quad x > 0; \tag{6}$$

$$\begin{cases} \tau_{yz}^{(3)}(x,-h,t) = \tau_{yz}^{(4)}(x,-h,t), \\ \phi^{(3)}(x,-h,t) = \phi^{(4)}(x,-h,t), \quad x > 0; \\ D_y^{(3)}(x,-h,t) = D_y^{(4)}(x,-h,t), \end{cases} \tag{7}$$

$$\tau_{yz}^{(1)}(x,h,t) = \tau_{yz}^{(2)}(x,h,t) = -\tau_0, \quad 0 \leqslant x \leqslant a; \tag{8}$$

$$\tau_{yz}^{(3)}(x,-h,t) = \tau_{yz}^{(4)}(x,-h,t) = -\tau_0 e^{-i\omega(2h/c_2)}, \quad 0 \leqslant x \leqslant b; \tag{9}$$

$$w^{(1)}(x,h,t) = w^{(2)}(x,h,t), \quad x > a; \tag{10}$$

$$w^{(3)}(x,-h,t) = w^{(4)}(x,-h,t), \quad x > b. \tag{11}$$

Assume that the solutions of Eqs. (1) and (2) are as follows:

$$\begin{cases} w^{(1)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_1(s)e^{-\gamma^{(1)}y}\cos(sx)ds, \\ \phi^{(1)}(x,y,t) = \dfrac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}w^{(1)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_1(s)e^{-sy}\cos(sx)ds; \end{cases} \tag{12}$$

$$\begin{cases} w^{(2)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty [A_2(s)e^{-\gamma^{(2)}y} + B_2(s)e^{\gamma^{(2)}y}]\cos(sx)ds, \\ \phi^{(2)}(x,y,t) = \dfrac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}w^{(2)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty [C_2(s)e^{-sy} + D_2(s)e^{sy}]\cos(sx)ds; \end{cases} \tag{13}$$

$$\begin{cases} w^{(3)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty [A_3(s)e^{\gamma^{(2)}y} + B_3(s)e^{-\gamma^{(2)}y}]\cos(sx)ds, \\ \phi^{(3)}(x,y,t) = \dfrac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}w^{(3)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty [C_3(s)e^{sy} + D_3(s)e^{-sy}]\cos(sx)ds; \end{cases} \tag{14}$$

$$\begin{cases} w^{(4)}(x,y,t) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_4(s)e^{\gamma^{(1)}y}\cos(sx)ds, \\ \phi^{(4)}(x,y,t) = \dfrac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}w^{(4)}(x,y,t) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_4(s)e^{sy}\cos(sx)ds, \end{cases} \tag{15}$$

where $(\gamma^{(k)})^2 = s^2 - \omega^2/c_k^2$, $c_k^2 = \mu^{(k)}/\rho^{(k)}$, $\mu^{(k)} = c_{44}^{(k)} + (e_{15}^{(k)})^2/\varepsilon_{11}^{(k)}$. $A_k(s)$, $B_k(s)$, $C_k(s)$, $D_k(s)$ are unknown functions.

So, substituting Eqs. (12) ~ (15) into Eqs. (3) and (4), we obtain

$$\begin{cases} \tau_{yz}^{(1)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \{\mu^{(1)}\gamma^{(1)}A_1(s)e^{-\gamma^{(1)}y} + e_{15}^{(1)}sC_1(s)e^{-sy}\}\cos(sx)\,ds, \\ D_y^{(1)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(1)}sC_1(s)e^{-sy}\cos(sx)\,ds; \end{cases} \tag{16}$$

$$\begin{cases} \tau_{yz}^{(2)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \{\mu^{(2)}\gamma^{(2)}[A_2(s)e^{-\gamma^{(2)}y} - B_2(s)e^{\gamma^{(2)}y}] + \\ \qquad e_{15}^{(2)}s[C_2(s)e^{-sy} - D_2(s)e^{sy}]\}\cos(sx)\,ds, \\ D_y^{(2)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(2)}s[C_2(s)e^{-sy} - D_2(s)e^{sy}]\cos(sx)\,ds; \end{cases} \tag{17}$$

$$\begin{cases} \tau_{yz}^{(3)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \{\mu^{(2)}\gamma^{(2)}[A_3(s)e^{\gamma^{(2)}y} - B_3(s)e^{-\gamma^{(2)}y}] + \\ \qquad e_{15}^{(2)}s[C_3(s)e^{sy} - D_3(s)e^{-sy}]\}\cos(sx)\,ds, \\ D_y^{(3)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(2)}s[C_3(s)e^{sy} - D_3(s)e^{-sy}]\cos(sx)\,ds; \end{cases} \tag{18}$$

$$\begin{cases} \tau_{yz}^{(4)} = \dfrac{2}{\pi}\displaystyle\int_0^\infty \{\mu^{(1)}\gamma^{(1)}A_4(s)e^{\gamma^{(1)}y} + e_{15}^{(1)}sC_4(s)e^{sy}\}\cos(sx)\,ds, \\ D_y^{(4)} = -\dfrac{2}{\pi}\displaystyle\int_0^\infty \varepsilon_{11}^{(1)}sC_4(s)e^{sy}\cos(sx)\,ds. \end{cases} \tag{19}$$

To solve the problem, the gap functions of the crack surface displacements are defined as follows:

$$f_1(x) = w^{(1)}(x,h,t) - w^{(2)}(x,h,t), \tag{20}$$

$$f_2(x) = w^{(3)}(x,-h,t) - w^{(4)}(x,-h,t). \tag{21}$$

Substituting Eqs. (12) ~ (15) into Eqs. (20) ~ (21), and applying Fourier cosine transforms (a superposed bar indicates the Fourier cosine transforms throughout the paper) with the boundary conditions (5), (7), (10) and (11), it can be obtained

$$\bar{f}_1(s) = A_1(s)e^{-\gamma^{(1)}h} - A_2(s)e^{-\gamma^{(2)}h} - B_2(s)e^{\gamma^{(2)}h}, \tag{22}$$

$$\bar{f}_2(s) = A_3(s)e^{-\gamma^{(2)}h} + B_3(s)e^{\gamma^{(2)}h} - A_4(s)e^{-\gamma^{(1)}h}, \tag{23}$$

$$\frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}A_1(s)e^{-\gamma^{(1)}h} + C_1(s)e^{-sh} - \frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}[A_2(s)e^{-\gamma^{(2)}h} + B_2(s)e^{\gamma^{(2)}h}] - [C_2(s)e^{-sh} + D_2(s)e^{sh}] = 0, \tag{24}$$

$$\frac{e_{15}^{(2)}}{\varepsilon_{11}^{(2)}}[A_3(s)e^{-\gamma^{(2)}h} + B_3(s)e^{\gamma^{(2)}h}] + [C_3(s)e^{-sh} + D_3(s)e^{sh}] - \frac{e_{15}^{(1)}}{\varepsilon_{11}^{(1)}}A_4(s)e^{-\gamma^{(1)}h} - C_4(s)e^{-sh} = 0. \tag{25}$$

By applying Fourier cosine transforms to Eqs. (16) ~ (19) with boundary conditions (5) ~ (11), it can be obtained

$$\mu^{(1)}\gamma^{(1)}A_1(s)e^{-\gamma^{(1)}h} + e_{15}^{(1)}sC_1(s)e^{-sh} = \mu^{(2)}\gamma^{(2)}[A_2(s)e^{-\gamma^{(2)}h} - B_2(s)e^{\gamma^{(2)}h}] + e_{15}^{(2)}s[C_2(s)e^{-sh} - D_2(s)e^{sh}], \tag{26}$$

$$\mu^{(2)}\gamma^{(2)}[A_3(s)e^{-\gamma^{(2)}h} - B_3(s)e^{\gamma^{(2)}h}] + e_{15}^{(2)}s[C_3(s)e^{-sh} - D_3(s)e^{sh}]$$
$$= \mu^{(1)}\gamma^{(1)}A_4(s)e^{-\gamma^{(1)}h} + e_{15}^{(1)}sC_4(s)e^{-sh}, \tag{27}$$

$$\varepsilon_{11}^{(1)}C_1(s)e^{-sh} = \varepsilon_{11}^{(2)}[C_2(s)e^{-sh} - D_2(s)e^{sh}],$$
$$\varepsilon_{11}^{(2)}[C_3(s)e^{-sh} - D_3(s)e^{sh}] = \varepsilon_{11}^{(1)}C_4(s)e^{-sh}, \tag{28}$$

$$A_2(s) + B_2(s) = A_3(s) + B_3(s),\ A_2(s) - B_2(s) = -A_3(s) + B_3(s), \tag{29}$$

$$C_2(s) + D_2(s) = C_3(s) + D_3(s),\ C_2(s) - D_2(s) = -C_3(s) + D_3(s). \tag{30}$$

By solving twelve Eqs. (22) ~ (30) with twelve unknown functions $A_k(s)$, $B_k(s)$, $C_k(s)$, $D_k(s)$, and substituting the solutions into Eqs. (16) and (19) and applying the boundary conditions (5), (7) ~ (9), it can be obtained

$$\frac{2}{\pi}\int_0^\infty \bar{f}_1(s)\cos(sx)ds = 0, \qquad x > a, \tag{31}$$

$$\frac{2}{\pi}\int_0^\infty \bar{f}_2(s)\cos(sx)ds = 0, \qquad x > b, \tag{32}$$

$$\frac{2}{\pi}\int_0^\infty s[\alpha(s)\bar{f}_1(s) + \beta(s)\bar{f}_2(s)]\cos(sx)ds = -\tau_0, \qquad 0 \leqslant x \leqslant a, \tag{33}$$

$$\frac{2}{\pi}\int_0^\infty s[\beta(s)\bar{f}_1(s) + \alpha(s)\bar{f}_2(s)]\cos(sx)ds = -\tau_0 e^{-i\omega(2h/c_2)}, \qquad 0 \leqslant x \leqslant b, \tag{34}$$

where $\alpha(s)$ and $\beta(s)$ are known functions and given in Appendix.

2 Solutions of the Dual Integral Equations

The set of dual integral Eqs. (31) ~ (34) may be solved by use of the Schmidt method. The gap functions of the crack surface displacement are represented by the following series:

$$f_1(x) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{a}\right)\left(1 - \frac{x^2}{a^2}\right)^{1/2}, \quad \text{for } 0 \leqslant x \leqslant a,\ y = h, \tag{35a}$$

$$f_2(x) = \sum_{n=1}^{\infty} b_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{b}\right)\left(1 - \frac{x^2}{b^2}\right)^{1/2}, \quad \text{for } 0 \leqslant x \leqslant b,\ y = -h, \tag{35b}$$

where a_n and b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial (Gradshteyn and Ryzhik[11]). The Fourier cosine transforms of Eq. (35) are (Erdelyi[12])

$$\bar{f}_1(s) = \frac{1}{s}\sum_{n=1}^{\infty} a_n G_n J_{2n-1}(sa), \quad \bar{f}_2(s) = \frac{1}{s}\sum_{n=1}^{\infty} b_n G_n J_{2n-1}(sb), \tag{36}$$

in which $G_n = 2\sqrt{\pi}(-1)^{n-1}[\Gamma(2n - 1/2)/(2n-2)!]$, $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eq. (36) into Eqs. (31) ~ (34), Eqs. (31) and (32) have been automatically satisfied, respectively. After integration with respect to x in $[0, x]$, where $0 < x < a$ in Eq. (33) and $0 < x < b$ in Eq. (34), the Eqs. (33) and (34) are reduced to

$$\sum_{n=1}^{\infty} a_n G_n \int_0^\infty \frac{1}{s}\alpha(s)J_{2n-1}(sa)\sin(sx)ds +$$
$$\sum_{n=1}^{\infty} b_n G_n \int_0^\infty \frac{1}{s}\beta(s)J_{2n-1}(sb)\sin(sx)ds = -\frac{\pi\tau_0}{2}x, \qquad 0 < x < a, \tag{37a}$$

$$\sum_{n=1}^{\infty} a_n G_n \int_0^\infty \frac{1}{s}\beta(s)J_{2n-1}(sa)\sin(sx)ds +$$

$$\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty} \frac{1}{s} \alpha(s) J_{2n-1}(sb) \sin(sx) ds = -\frac{\pi \tau_0}{2} x e^{-i\omega(2h/c_2)}, \quad 0 < x < b, \tag{37b}$$

Eqs. (37) can now be solved for the coefficients a_n and b_n by the Schmidt method (Zhou and Wang[7], Itou[13], Zhou, Han and Du[14]). The method is omitted in the present work.

3 Field Intensity Factors

The coefficients a_n and b_n are known, so that the entire stress field and the electric displacement field can be obtained. From the form of the stress field and the electric displacement, the singular parts of the stress field and the singular part of the electric displacement can be expressed respectively as follows:

$$\tau^{(1)} = -\frac{2\alpha_c}{\pi} \sum_{n=1}^{\infty} a_n G_n H_n(x), \quad D^{(1)} = -\frac{2\delta_c}{\pi} \sum_{n=1}^{\infty} a_n G_n H_n(x), \quad \text{for} \quad x > a, \tag{38a}$$

$$\tau^{(4)} = -\frac{2\alpha_c}{\pi} \sum_{n=1}^{\infty} b_n G_n L_n(x), \quad D^{(4)} = -\frac{2\delta_c}{\pi} \sum_{n=1}^{\infty} b_n G_n L_n(x), \quad \text{for} \quad x > b, \tag{38b}$$

where $H_n(x) = \dfrac{(-1)^{n-1} a^{2n-1}}{\sqrt{x^2 - a^2}[x + \sqrt{x^2 - a^2}]^{2n-1}}$, $L_n(x) = \dfrac{(-1)^{n-1} b^{2n-1}}{\sqrt{x^2 - b^2}[x + \sqrt{x^2 - b^2}]^{2n-1}}$,

where $\gamma(s)$, $\delta(s)$, α_c and δ_c are given in Appendix.

We obtain the stress intensity factors K_a and K_b as

$$K_a = \lim_{x \to a^+} \sqrt{2\pi(x - a)}\, \tau^{(1)} = -\frac{4\alpha_c}{\sqrt{a}} \sum_{n=1}^{\infty} a_n \frac{\Gamma(2n - 1/2)}{(2n - 2)!}, \tag{39}$$

$$K_b = \lim_{x \to b^+} \sqrt{2\pi(x - b)}\, \tau^{(4)} = -\frac{4\alpha_c}{\sqrt{b}} \sum_{n=1}^{\infty} b_n \frac{\Gamma(2n - 1/2)}{(2n - 2)!}. \tag{40}$$

We obtain the electric displacement intensity factors D_a and D_b as

$$D_a = \lim_{x \to a^+} \sqrt{2\pi(x - a)}\, D^{(1)} = -\frac{4\delta_c}{\sqrt{a}} \sum_{n=1}^{\infty} a_n \frac{\Gamma(2n - 1/2)}{(2n - 2)!} = \frac{\delta_c}{\alpha_c} K_a, \tag{41}$$

$$D_b = \lim_{x \to b^+} \sqrt{2\pi(x - b)}\, D^{(4)} = -\frac{4\delta_c}{\sqrt{b}} \sum_{n=1}^{\infty} b_n \frac{\Gamma(2n - 1/2)}{(2n - 2)!} = \frac{\delta_c}{\alpha_c} K_b. \tag{42}$$

4 Numerical Calculations and Discussion

We carried out numerical calculations for the piezoelectric ceramic. In the layered structure, materials 1 and 2 are piezoelectric ceramic PZT-4 and PZT-5H as in Fig. 1, respectively. The material constants are listed in Table 1. From Refs. [9, 10], it can be seen that the Schmidt method performs satisfactorily if the first six terms of the infinite series (37) are retained. The dimensionless stress intensity factors (K_a, K_b) and the electric displacement intensity factors (D_a, D_b) are calculated numerically. The results are shown in Figs. 2 ~ 14.

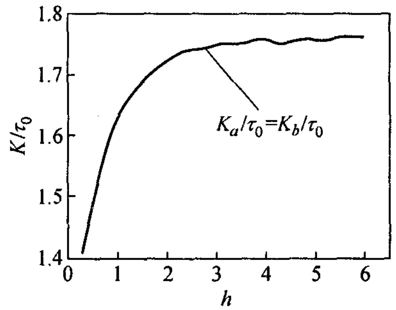

Fig. 2 The stress intensity factor versus h ($\omega/c_1 = 0$, $a = b = 1.0$, PZT-4/PZT-4/PZT-4)

Table 1 Material constants of piezoelectric ceramic

Materials	$c_{44}/(\mathrm{N/m^2})$	$e_{15}/(\mathrm{C/m^2})$	$\varepsilon_{11}/(\mathrm{C/V\cdot m})$	$\rho/(\mathrm{kg/m^3})$
PZT-4	2.56×10^{10}	12.7	64.6×10^{10}	7 500
PZT-5H	2.3×10^{10}	17.0	150.4×10^{10}	7 500

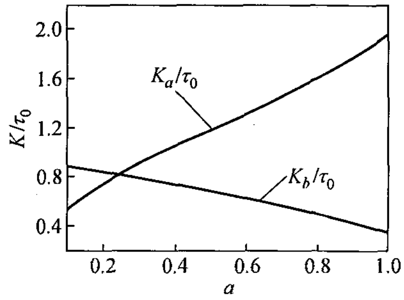

Fig.3 The stress intensity factor versus $a(h = 0.5, \omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

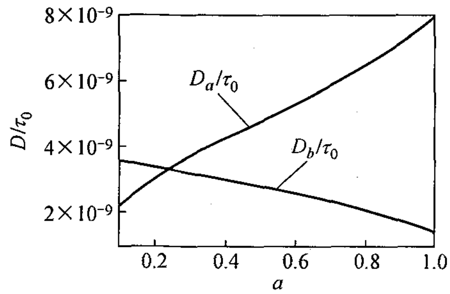

Fig.4 The electric displacement intensity factor versus a ($h = 0.5$, $\omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

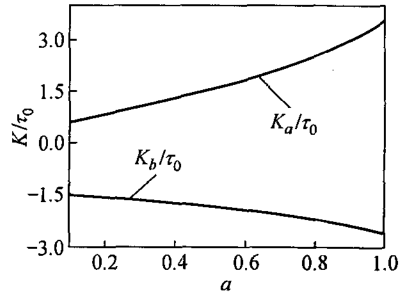

Fig.5 The stress intensity factor versus $a(h = 1.0, \omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

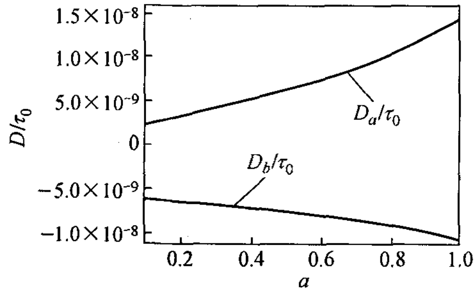

Fig.6 The electric displacement intensity factor versus $a(h = 1.0$, $\omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

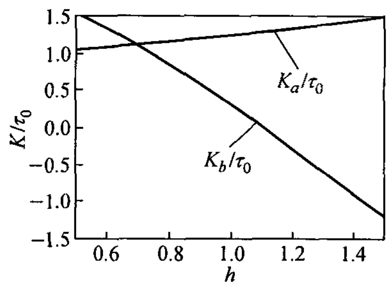

Fig.7 The stress intensity factor versus $h(a = 0.5, \omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

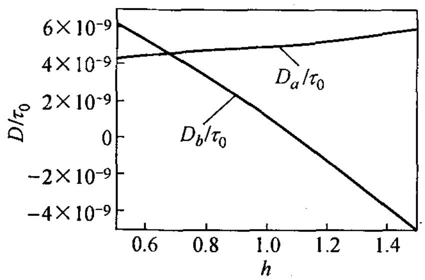

Fig.8 The electric displacement intensity factor versus h ($a = 0.5, \omega/c_1 = 0.5$, $b = 1.0$, PZT-4/PZT-5H/PZT-4)

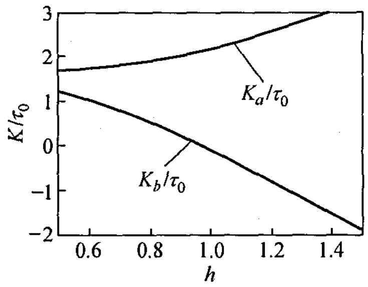

Fig.9 The stress intensity factor versus $h(a = 1.0, \omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

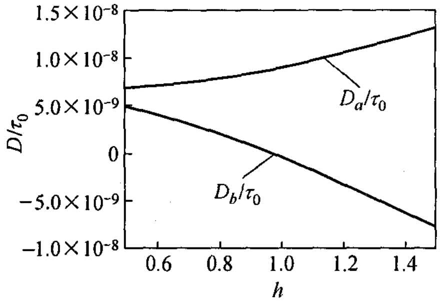

Fig.10 The electric displacement intensity factor versus $h(a = 1.0, \omega/c_1 = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

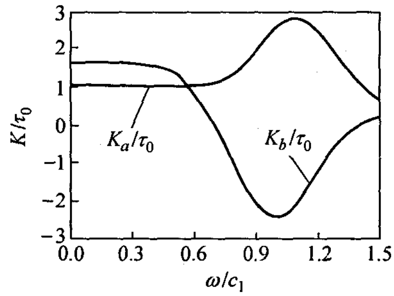

Fig.11 The stress intensity factor versus $\omega/c_1(h = 0.5, a = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

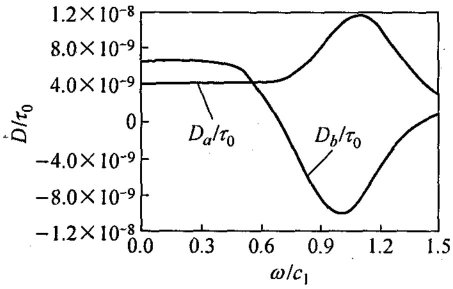

Fig.12 The electric displacement intensity factor versus $\omega/c_1(h = 0.5, a = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

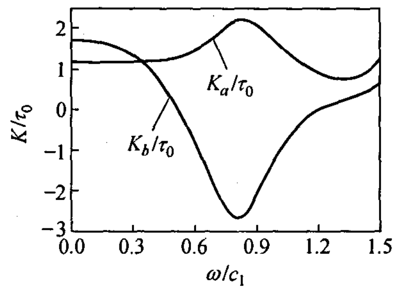

Fig.13 The stress intensity factor versus $\omega/c_1(h = 1.0, a = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

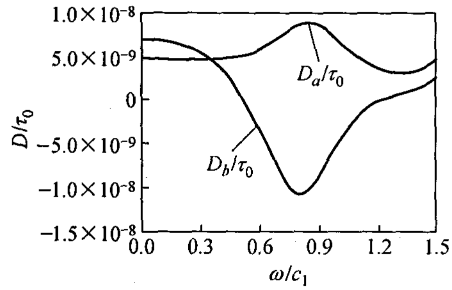

Fig.14 The electric displacement intensity factor versus $\omega/c_1(h = 1.0, a = 0.5, b = 1.0$, PZT-4/PZT-5H/PZT-4)

The following observations are very significant:

(i) The results of the present problem are the same as those in Ref.[7] when $a = b$ and $\omega = 0$ as shown in Fig.2.

(ii) The results in Figs.3 ~ 6 indicate that K_a increases with increase of the length of crack 1. However, K_b decreases with increase of the length of crack 1. Especially, when $a = 0.24$, the curves intersect in Fig.3, which reveals that K_a can be equal to K_b in suitable cases even for $a \neq b$. This behavior is caused by the different mechanical boundary conditions at cracks 1 and 2.

(iii) It can be seen that K_a increases with increase of the distance between the parallel cracks. This phenomenon is called crack shielding effect (Ratwani[15]). However, K_b decreases with increase of the distance between the parallel cracks as shown in Figs.7 ~ 10.

(iv) It can be obtained that K_a increases extremely with increase of the length of crack 1. However, the stress intensity factor K_b decreases with increase of the distance between the parallel cracks as shown in Figs.3 ~ 10. From this, it can be obtained that the effects of the length of crack 1 on K_a is greater than on K_b, and the effects of the distance between the parallel cracks on K_b is greater than on K_a.

(v) From the results in Figs.11 ~ 14, it can be shown that the stress intensity factor K_b becomes negative value with increase of frequency of incident wave. This phenomenon is caused by the boundary condition at crack 2.

(vi) From Eqs.(41) ~ (42), it can be obtained that the tendency of D_a and D_b are similar with trend of K_a and K_b, respectively.

From the results, it can be seen that the dynamic stress intensity factors and the electric displacement intensity factors depend on the crack length, the frequency of incident wave and the distance between two cracks.

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Appendix

$$(\gamma^{(1)})^2 = s^2 - \omega^2/c_1{}^2,\ c_1{}^2 = \mu^{(1)}/\rho^{(1)},$$

$$\mu^{(1)} = c_{44}^{(1)} + (e_{15}^{(1)})^2/\varepsilon_{11}^{(1)},\ (\gamma_N^{(1)})^2 = 1 - \omega^2/(c_1{}^2 s^2);$$

$$(\gamma^{(2)})^2 = s^2 - \omega^2/c_2{}^2,\ c_2{}^2 = \mu^{(2)}/\rho^{(2)},$$

$$\mu^{(2)} = c_{44}^{(2)} + (e_{15}^{(2)})^2/\varepsilon_{11}^{(2)},\ (\gamma_N^{(2)})^2 = 1 - \omega^2/(c_2{}^2 s^2);$$

$$H_1 = 1 + \mathrm{e}^{-4h\gamma^{(2)}},\ H_2 = 1 - \mathrm{e}^{-4h\gamma^{(2)}},\ H_3 = 1 + \mathrm{e}^{-4hs},\ H_4 = 1 - \mathrm{e}^{-4hs};$$

$$R_1 = 2H_1H_4(e_{15}^{(1)})^3 e_{15}^{(2)}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^3,$$

$$R_2 = -2H_1H_4 e_{15}^{(1)} e_{15}^{(2)}\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^3,$$

$$R_3 = -H_1H_4(e_{15}^{(1)})^2\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(2)})^2((e_{15}^{(2)})^2(\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 - 2\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)})),$$

$$R_4 = H_1H_4\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2(2(e_{15}^{(2)})^2\varepsilon_{11}^{(1)} - \gamma_N^{(1)}\mu_1((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2)),$$

$$R_5 = 2H_1H_3\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^2\varepsilon_{11}^{(2)}((e_{15}^{(2)})^2(\varepsilon_{11}^{(1)})^2 - e_{15}^{(1)}e_{15}^{(2)}\varepsilon_{11}^{(1)}\varepsilon_{11}^{(2)} +$$
$$(\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 - \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)})),$$

$$R_6 = -H_2H_4(e_{15}^{(2)})^4\gamma_N^{(1)}\mu_1(\varepsilon_{11}^{(1)})^4,\ R_7 = 2H_2H_4 e_{15}^{(1)}(e_{15}^{(2)})^3\gamma_N^{(1)}\mu_1(\varepsilon_{11}^{(1)})^3\varepsilon_{11}^{(2)},$$

$$R_8 = -H_2H_4(e_{15}^{(2)})^2\gamma_N^{(1)}\mu_1(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 - \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}),$$

$$R_9 = H_2H_4(\gamma_N^{(2)})^2\mu_2{}^2\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^2(\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2) - (e_{15}^{(1)})^2(\varepsilon_{11}^{(2)})^2),$$

$$R_{10} = H_2H_3(\varepsilon_{11}^{(1)})^2\varepsilon_{11}^{(2)}((e_{15}^{(2)})^2(\gamma_N^{(1)})^2(\mu_1)^2(\varepsilon_{11}^{(1)})^2 + (\gamma_N^{(2)})^2\mu_2{}^2(\varepsilon_{11}^{(2)})^2((e_{15}^{(1)})^2 -$$
$$2\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)})),$$

$$R_{11} = -8\mathrm{e}^{-2h(\gamma^{(2)}+s)}e_{15}^{(2)}\gamma_N^{(1)}\gamma_N^{(2)}\mu_1\mu_2(\varepsilon_{11}^{(1)})^3\varepsilon_{11}^{(2)}(e_{15}^{(2)}\varepsilon_{11}^{(1)} - e_{15}^{(1)}\varepsilon_{11}^{(2)}),$$

$$R = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 + R_{10} + R_{11};$$

$$S_1 = 2\mathrm{e}^{-2h\gamma^{(2)}}\mathrm{e}^{-4hs}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^2(-e_{15}^{(1)}e_{15}^{(2)}\varepsilon_{11}^{(1)} + \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(1)} - \varepsilon_{11}^{(2)}) + (e_{15}^{(1)})^2(\varepsilon_{11}^{(2)})^2),$$

$$S_2 = -2\mathrm{e}^{-4h\gamma^{(2)}}\mathrm{e}^{-2hs}(\varepsilon_{11}^{(1)})^2\varepsilon_{11}^{(2)}(e_{15}^{(2)}\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)} - e_{15}^{(1)}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(2)})^2,$$

$$S_3 = 2\mathrm{e}^{-2hs}(\varepsilon_{11}^{(1)})^2\varepsilon_{11}^{(2)}(e_{15}^{(2)}\gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)} + e_{15}^{(1)}\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(2)})^2,$$

$$S_4 = -2\mathrm{e}^{-2h\gamma^{(2)}}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(2)})^2(e_{15}^{(1)}e_{15}^{(2)}\varepsilon_{11}^{(1)} - (e_{15}^{(1)})^2\varepsilon_{11}^{(2)} + \gamma_N^{(1)}\mu_1\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)}))^2,$$

$$S = S_1 + S_2 + S_3 + S_4;$$

$$T_1 = H_2H_4(e_{15}^{(1)})^3\gamma_N^{(1)}\mu_1(\varepsilon_{11}^{(1)})^4\varepsilon_{11}^{(2)},\ T_2 = H_1H_4(e_{15}^{(1)})^3\gamma_N^{(2)}\mu_2\varepsilon_{11}^{(1)}(\varepsilon_{11}^{(2)})^4,$$

$$T_3 = -H_2 e_{15}^{(1)}(\gamma_N^{(2)})^2\mu_2{}^2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^3(\varepsilon_{11}^{(1)}H_3 - \varepsilon_{11}^{(2)}H_4),$$

$$T_4 = -2H_1H_4(e_{15}^{(1)})^2 e_{15}^{(2)}\gamma_N^{(2)}\mu_2(\varepsilon_{11}^{(1)})^2(\varepsilon_{11}^{(2)})^3,$$

$T_5 = -H_2 e_{15}^{(1)}(\gamma_N^{(1)})^2 \mu_1{}^2(\varepsilon_{11}^{(1)})^3(\varepsilon_{11}^{(2)})^2(\varepsilon_{11}^{(1)} H_3 - \varepsilon_{11}^{(2)} H_4)$,

$T_6 = H_2 H_4 (e_{15}^{(1)})^2 e_{15}^{(2)} \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3$,

$T_7 = -(H_3 H_1 - 4e^{-2h\gamma^{(2)}} e^{-2hs}) e_{15}^{(2)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^4 (\varepsilon_{11}^{(2)})^2$,

$T_8 = -H_1 H_4 e_{15}^{(2)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^3$,

$T_9 = -2H_2 H_4 e_{15}^{(1)} (e_{15}^{(2)})^2 \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^2$,

$T_{10} = H_1 H_4 e_{15}^{(1)} (e_{15}^{(2)})^2 \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^2$,

$T_{11} = -(H_3 H_1 + 4e^{-2h\gamma^{(2)}} e^{-2hs}) e_{15}^{(1)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)}))^3$,

$T_{12} = -H_1 H_4 e_{15}^{(1)} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^4$,

$T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11} + T_{12}$;

$U_1 = 2e^{-2h\gamma^{(2)}} e_{15}^{(1)} (e_{15}^{(2)})^2 \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^3$,

$U_2 = 2e^{-4h\gamma^{(2)}} e^{-2hs} (\gamma_N^{(1)} \mu_1 - \gamma_N^{(2)} \mu_2)(\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^2 (e_{15}^{(2)} \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} - e_{15}^{(1)} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(2)})$,

$U_3 = -2e^{-2hs} (\gamma_N^{(1)} \mu_1 + \gamma_N^{(2)} \mu_2)(\varepsilon_{11}^{(1)})^3 (\varepsilon_{11}^{(2)})^2 (e_{15}^{(2)} \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} + e_{15}^{(1)} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(2)})$,

$U_4 = -2e^{-2h\gamma^{(2)}} e^{-4hs} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^2 (e_{15}^{(2)} \varepsilon_{11}^{(1)} + e_{15}^{(1)} \varepsilon_{11}^{(2)})(e_{15}^{(1)} e_{15}^{(2)} s\varepsilon_{11}^{(1)} - (e_{15}^{(1)})^2 s\varepsilon_{11}^{(2)})$,

$U_5 = 2e^{-2h\gamma^{(2)}} e_{15}^{(1)} \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^3 ((e_{15}^{(1)})^2 \varepsilon_{11}^{(2)} - \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)}))$,

$U_6 = 2e^{-2h\gamma^{(2)}} e_{15}^{(2)} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 (-2(e_{15}^{(1)})^2 \varepsilon_{11}^{(2)} + \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(1)} + \varepsilon_{11}^{(2)}))$,

$U_7 = -2e^{-2h\gamma^{(2)}} e^{-4hs} \gamma_N^{(1)} \gamma_N^{(2)} \mu_1 \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 (e_{15}^{(2)} \varepsilon_{11}^{(1)} + e_{15}^{(1)} \varepsilon_{11}^{(2)})(\varepsilon_{11}^{(2)} - \varepsilon_{11}^{(1)})$,

$U = U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_7$;

$V_1 = H_2 H_4 (e_{15}^{(2)})^2 (\varepsilon_{11}^{(1)})^3 (e_{15}^{(2)} \varepsilon_{11}^{(1)} - 4e_{15}^{(1)} \varepsilon_{11}^{(2)})$,

$V_2 = 2H_2 H_4 (e_{15}^{(2)})^2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 (3(e_{15}^{(1)})^2 - \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)})$,

$V_3 = 4H_2 H_4 e_{15}^{(1)} e_{15}^{(2)} \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^3 (-(e_{15}^{(1)})^2 + \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)})$,

$V_4 = H_2 H_4 (\varepsilon_{11}^{(2)})^2 ((e_{15}^{(1)})^4 (\varepsilon_{11}^{(2)})^2 - 2(e_{15}^{(1)})^2 \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^2 +$
$(\gamma_N^{(1)})^2 (\mu_1)^2 (\varepsilon_{11}^{(1)})^2 ((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2))$,

$V_5 = H_2 H_4 (\gamma_N^{(2)})^2 (\mu_2)^2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 ((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2)$,

$V_6 = 8e^{-2h\gamma^{(2)}} e^{-2hs} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^2 \varepsilon_{11}^{(2)} (e_{15}^{(2)} \varepsilon_{11}^{(1)} + e_{15}^{(1)} \varepsilon_{11}^{(2)})^2$,

$V_7 = -2H_2 H_3 e_{15}^{(2)} \gamma_N^{(1)} \mu_1 (\varepsilon_{11}^{(1)})^3 \varepsilon_{11}^{(2)} (e_{15}^{(2)} \varepsilon_{11}^{(1)} - 2e_{15}^{(1)} \varepsilon_{11}^{(2)})$,

$V_8 = 2H_2 H_3 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3 (-(e_{15}^{(1)})^2 \gamma_N^{(1)} \mu_1 + (\gamma_N^{(1)})^2 (\mu_1)^2 \varepsilon_{11}^{(1)} + (\gamma_N^{(2)})^2 (\mu_2)^2 \varepsilon_{11}^{(1)})$,

$V_9 = -2H_1 H_4 e_{15}^{(2)} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 (e_{15}^{(2)} \varepsilon_{11}^{(1)} - 2e_{15}^{(1)} \varepsilon_{11}^{(2)})$,

$V_{10} = 2H_1 H_4 \gamma_N^{(2)} \mu_2 \varepsilon_{11}^{(1)} (\varepsilon_{11}^{(2)})^2 (-(e_{15}^{(1)})^2 (\varepsilon_{11}^{(2)})^2 + \gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)} ((\varepsilon_{11}^{(1)})^2 + (\varepsilon_{11}^{(2)})^2))$,

$V_{11} = -2H_1 H_3 e_{15}^{(2)} \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^3 \varepsilon_{11}^{(2)} (e_{15}^{(2)} \varepsilon_{11}^{(1)} - 2e_{15}^{(1)} \varepsilon_{11}^{(2)})$,

$V_{12} = -2H_1 H_3 \gamma_N^{(2)} \mu_2 (\varepsilon_{11}^{(1)})^2 (\varepsilon_{11}^{(2)})^3 ((e_{15}^{(1)})^2 - 2\gamma_N^{(1)} \mu_1 \varepsilon_{11}^{(1)})$,

$V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8 + V_9 + V_{10} + V_{11} + V_{12}$;

$\alpha(s) = R/V,\ \beta(s) = S/V,\ \delta(s) = T/V,\ \gamma(s) = U/V,$

$\lim_{s\to\infty} \alpha(s) = \alpha_c,\ \lim_{s\to\infty} \beta(s) = \beta_c,\ \lim_{s\to\infty} \delta(s) = \delta_c.$